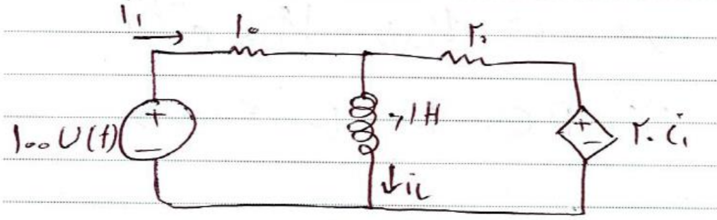
مثال حالت صفر * (۱)

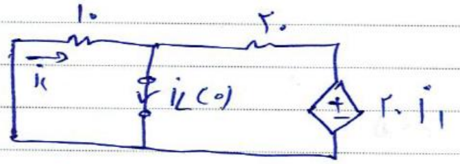
Subject:
Year: Month: Date:

ص ۲۱ سوال ۶.



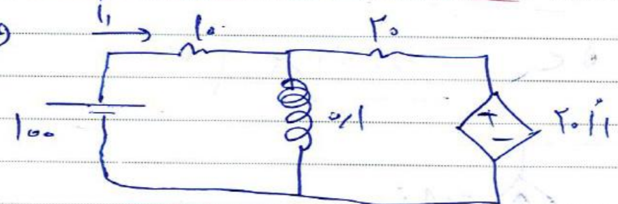

$i_L(t) = ?$ و $i_1(t) = ?$

$t<0 \Rightarrow$



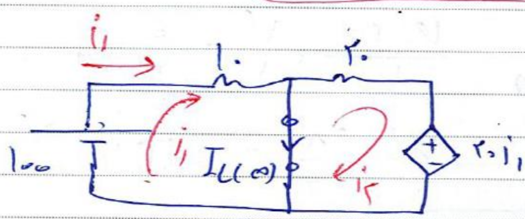

$i_L(0) = 0$

$t \geq 0 \Rightarrow$




حالت صفر $\Rightarrow$ $\boxed{i_L(t) = i_L(\infty)(1-e^{-t/\tau})}$

بدست آوردن : $I_L(\infty)$

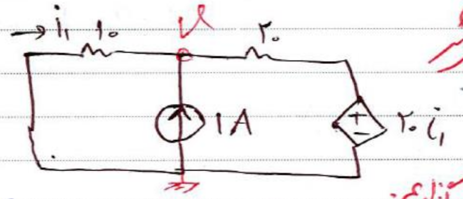



$-100 + 10 i_1 = 0 \Rightarrow i_1 = 10\,A$

$20 i_2 + 20 i_1 = 0 \Rightarrow i_2 = -10\,A$

$\boxed{i_L(\infty) = i_1 - i_2 = 20\,A}$ ✓

بدست آوردن $\tau$ :




منابع مستقل برابر

$R_N = R_{th} = R_{eq}$

چون منبع وابسته داریم یک منبع جریان 1A در مدار می‌گذاریم

$$\begin{cases} \dfrac{v}{10} - 1 + \dfrac{v - 20 i_1}{20} = 0 \\ i_1 = \dfrac{-v}{10} \end{cases} \Rightarrow \frac{v}{10} - 1 + \frac{v + 2v}{20} = 0$$

$$\Rightarrow 5v = 20 \Rightarrow v = 4\,V$$

$$R_{eq} = \frac{4}{1} = 4\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0/1}{4} = \frac{1}{40} \Rightarrow \frac{1}{\tau} = 40$$

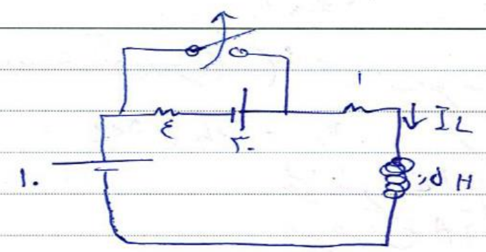
$$\boxed{i_L(t) = 20\left(1 - e^{-40t}\right)}$$

$$v_L(t) = L\frac{di}{dt} = 0/1\left(20\left(40 e^{-40t}\right)\right) = \boxed{80 e^{-40t}}$$

$$i_1(t) = \frac{100 - v_L(t)}{10} = \frac{100 - 80 e^{-40t}}{10} \Rightarrow \boxed{i_1(t) = 10 - 8 e^{-40t}}$$

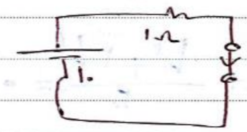
① * پاسخ کامل

ص ۳۰ سوال ۵۱ :



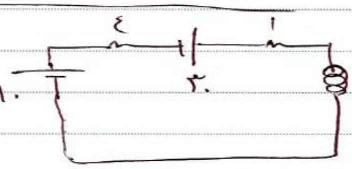

$I_L(t) = ?$

$t<0 \Rightarrow$

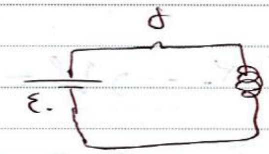



$I_L(0) = 10\,A$

$t>0 \Rightarrow$



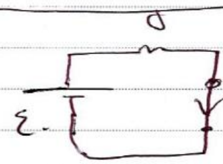

$\Rightarrow$




$I_L(\infty) = \frac{40}{5} = 8\,A$

پاسخ کامل $\Rightarrow I_L(t) = (I_L(0) - I_L(\infty))\,e^{-t/\tau} + I_L(\infty)$

بدست آوردن $I_L(\infty)$ :

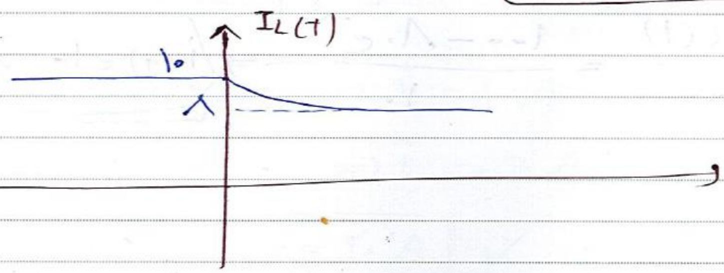



$I_L(\infty) = 8\,A$

بدست آوردن $\tau$ :

$$\tau = \frac{L}{R} = \frac{0.5}{5} = 0.1 \Rightarrow \frac{1}{\tau} = 10$$

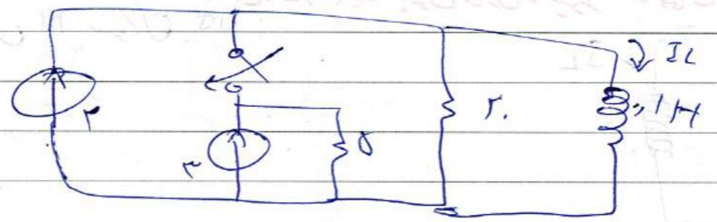
$$I_L(t) = (10-8)\,e^{-10t} + 8 \Rightarrow \boxed{I_L(t) = 8 + 2e^{-10t}}$$



① *پاسخ کامل

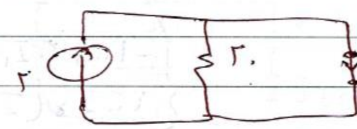
Date: .................... Subject: ....................

ص ۲۰ سوال ۵ :

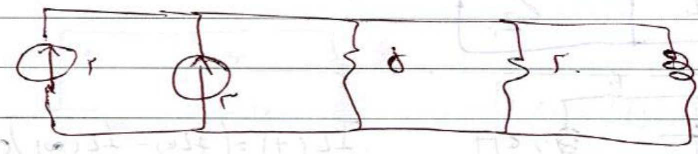



$I_L(t)=?$

$t<0 \Rightarrow$



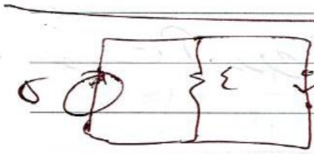

$t>0 \Rightarrow$




$\Longrightarrow$


پاسخ کامل $\Rightarrow I_L(t) = (I_L(0) - I_L(\infty))e^{-t/\tau} + I_L(\infty)$

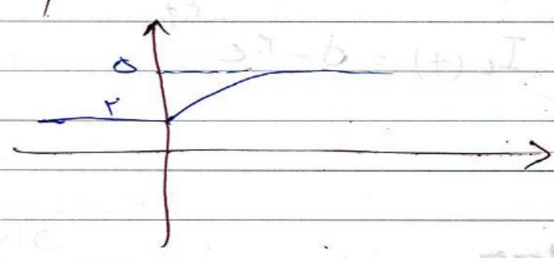
بدست آوردن $I_L(\infty)$ :




بدست آوردن $\tau$ : $\tau = \frac{L}{R} = \frac{0.1}{4} = \frac{1}{40} \Rightarrow \frac{1}{\tau} = 40$

$I_L(t) = (3-5)e^{-40t} + 5$

$\boxed{I_L(t) = 5 - 3e^{-40t}}$



۴ * پاسخ کامل $I_L(0^-) = I_L(0^+)$ زیرا که

$I_L(0^-) = I_L(0^+)$ همیشه مساوی است که باید $I_L(0^-)$ را پیدا کنیم و بعد از آن پیدا کردن $I_L(0^-) \Leftarrow I_L(0^+)$

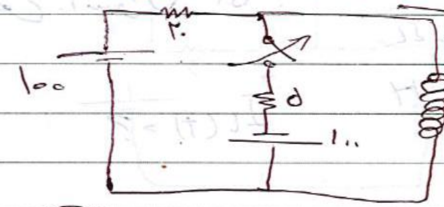
چون سلف با تغییر جریان مخالفت می‌کند

Date: برابر است

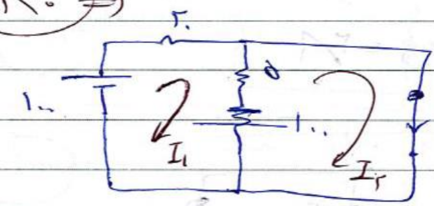
Subject: ولی $I_L(0) \neq I_L(\infty)$

$V_C(0^-) = V_C(0^+)$ چون خازن با تغییر ناگهانی ولتاژ مخالفت می‌کند.

ص ۲۱ سوال ۵۵ :




$t<0 \Rightarrow$

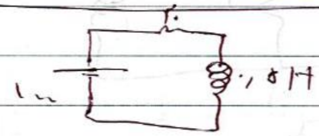



$I_{L(0)} = -15A$

$$\begin{cases} -100 + 20 I_1 + 5(I_1 - I_2) - 100 = 0 \\ 100 + 5(I_2 - I_1) = 0 \end{cases}$$

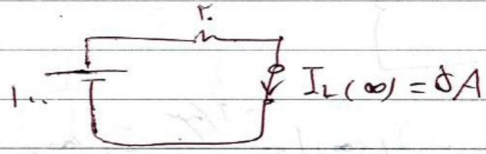
$$\Rightarrow I_2 = -15A$$

$t>0$




$$I_L(t) = \left(I_{L(0)} - I_{L(\infty)}\right) e^{-t/\tau} + I_{L(\infty)}$$

بدست آوردن $I_L(\infty)$ :




$I_L(\infty) = 5A$

$$\tau = \frac{L}{R} = \frac{0.5}{20} = \frac{1}{40} \Rightarrow \frac{1}{\tau} = 40$$

بدست آوردن $\tau$ :

$$I_L(t) = (-15 - 5) e^{-40t} + 5$$

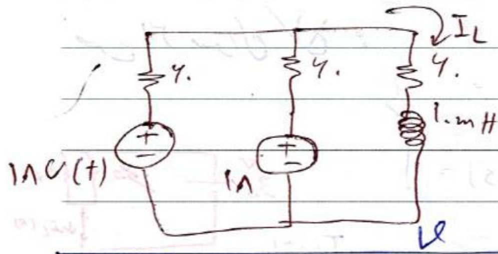
$$I_L(t) = 5 - 20 e^{-40t}$$

Raz

۴ * پاسخ کامل

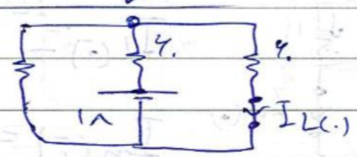
Date: ............ Subject: ............

ص ۲۱ سوال ۵۶



$I_L(t) = ?$

$t<0 \Rightarrow$

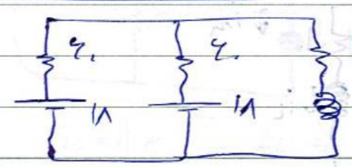


$$\frac{v}{60}+\frac{v-18}{60}+\frac{v}{60}=0$$

$$v = 6V$$

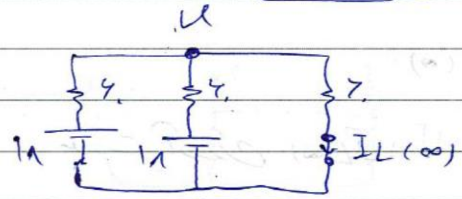
$$I_{L(0)} = \frac{6}{60} = 0.1A$$

$t>0 \Rightarrow$



$$I_L(t) = \left(I_{L(0)} - I_{L(\infty)}\right)e^{-t/\tau} + I_{L(\infty)}$$

بدست آوردن $I_{L(\infty)}$:

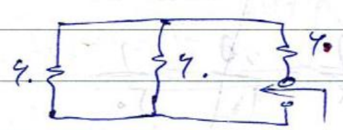


$$\frac{v-18}{60}+\frac{v-18}{60}+\frac{v}{60}=0$$

$$v = 12V$$

$$I_{L(\infty)} = \frac{12}{60} = 0.2A$$

بدست آوردن $\tau$:



$$R_{eq} = (60 \| 60) + 60 = 90\Omega$$

$$\tau = \frac{L}{R} = \frac{10mH}{90\Omega} \Rightarrow \frac{1}{\tau} = 9000$$

$$I_L(t) = (0.1 - 0.2)e^{-9000t} + 0.2$$

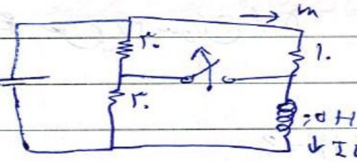
$$\boxed{I_L(t) = 0.2 - 0.1e^{-9000t}}$$

Raz

ط) پاسخ کامل

Date: Subject:

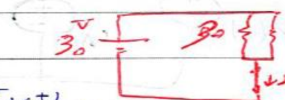
ص ۲۱ سوال ۵۷ :



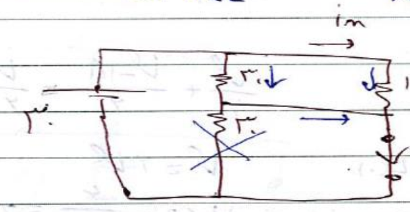
$i_n(0^-) =$

$i_n(0^+) =$

$i_n(4ms) =$



$t<0 \Rightarrow$



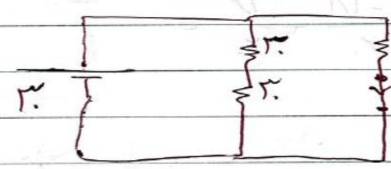
$I_L(0^-) = I_L(0^+) = \dfrac{30}{30 \| 10} = 4A$

$I_n(0^-) = \dfrac{30V}{10\Omega} = 3A$

$t>0 \Rightarrow$

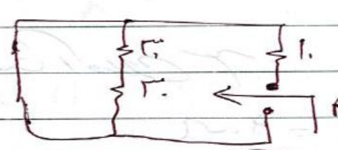
$i_L(t) = (I_L(0) - I_L(\infty))e^{-t/\tau} + I_L(\infty)$

بدست آوردن $I_L(\infty)$ :



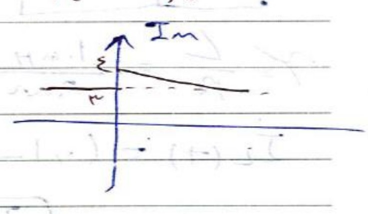
$I_L(\infty) = \dfrac{30}{10} = 3A$

بدست آوردن $\tau$ :



$R_{eq} = 10\Omega$ $\quad \tau = \dfrac{L}{R} = \dfrac{0.5}{10} = \dfrac{1}{20}$

$I_L(t) = e^{-20t} + 3$



$I_n(t) = I_L(t) \Rightarrow I_n(t) = e^{-20t} + 3$

Raz

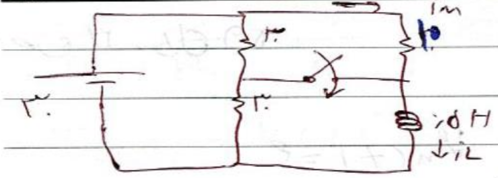
$I_n(0^+) = e^0 + 3 = 4A$

$I_n(4ms) = e^{-0.8} + 3 = 3.44A$

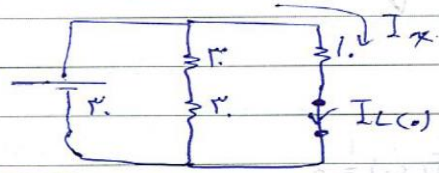
۶ *پاسخ کامل

Date: Subject:

ص ۳۱ سوال ۵۸ :

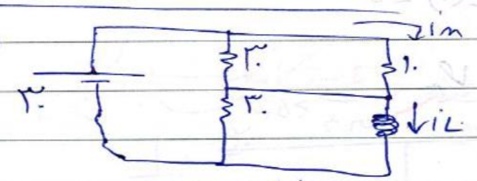


$t<0 \Rightarrow$



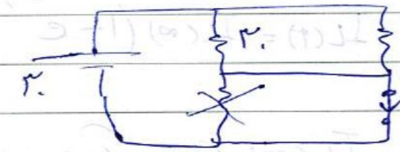
$I_{L(0)} = I_{x(0^-)} = \frac{30}{10} = 3A$

$t>0 \Rightarrow$



$I_{L(t)} = (I_{L(0)} - I_{L(\infty)})e^{-t/\tau} + I_{L(\infty)}$

جریان ماندگار $I_{L(\infty)}$ :



$I_{L(\infty)} = \frac{30}{30 \| 10} = 4A$

بدست آوردن $\tau$ :

$R_{eq} = 30 \| 30 \| 10 = 6\Omega \rightarrow \tau = \frac{L}{R} = \frac{0.5}{6} = \frac{1}{12}$

$$\boxed{I_{L(t)} = 4 - e^{-12t}}$$

$V_{L(t)} = L\frac{di}{dt} = 0.5(12e^{-12t}) = 6e^{-12t}$

$I_{m(t)} = \frac{30 - V_L(t)}{10\Omega} \Rightarrow \boxed{I_{m(t)} = 3 - 0.6e^{-12t}}$

$I_{m(0^+)} = 3 - 0.6e^{0} = 2.4A$

Raz

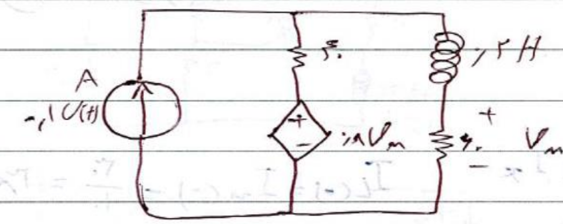
$I_m(40\,ms) = 3 - e^{-0.48} = 2.62A$

۳۷

② * حالت صفر

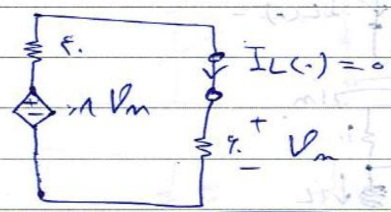


ص ۲۱ سوال ۵۹ :

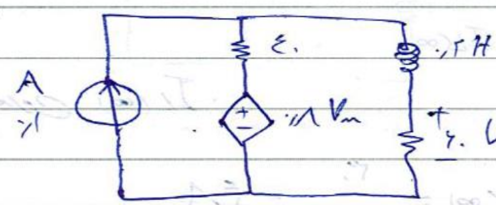



$V_m(t) = ?$

$t<0 \Rightarrow$

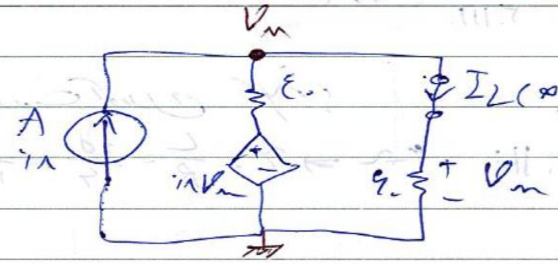



$t>0 \Rightarrow$




$I_L(t) = I_L(\infty)\left(1 - e^{-t/\tau}\right)$

برای بدست آوردن $I_L(\infty)$ :




$-0.1 + \frac{V_m - 0.8V_m}{40} + \frac{V_m}{60} = 0$

$2.6V_m = 12$

$V_m = 4.61\ V \rightarrow I_L(\infty) = \frac{4.61}{60} = 76\ mA$

برای بدست آوردن $\tau$ : $R_{eq}$ :

چون منابع مستقل برابر صفر شده و منبع ولتاژ ۱V را بجای سلف گذاشتیم


$\begin{cases} -0.8V_m + 40I + 1 + 60I = 0 \\ V_m = 60I \end{cases} \Rightarrow I = -\frac{1}{52}$

Raz

$R_{eq} = R_N = R_{th} = \frac{V}{I} = \frac{1}{\frac{1}{52}} = 52\,\Omega$

$\tau = \frac{L}{R} = \frac{0.2}{52} \Rightarrow \frac{1}{\tau} = 260$

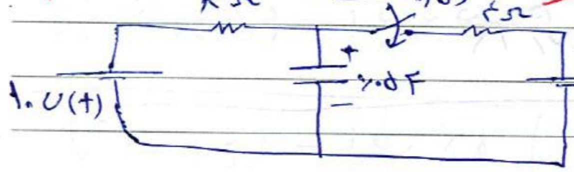
$$\boxed{I_L(t) = 76\times10^{-3}(1-e^{-260t})}$$

$V_m(t) = 60\, I_L(t)$

$$\boxed{V_m(t) = 4.61(1-e^{-260t})}$$ جواب

مثال ترکیبی: ② حالت صفر و پاسخ کامل (۲ کلید – ۲ مسئله)

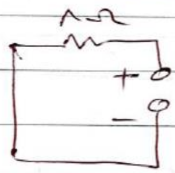
ص ۲۲ سوال ۶۷: اگر مبدا t را صفر باشد



$V_c(0.6s) = ?$

$V_c(0.8s) = ?$

یعنی ۰٫۳ s بعد از زدن کلید

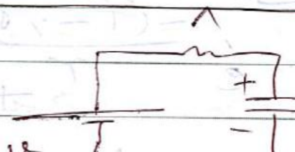


$t<0 \Rightarrow V_c(0) = 0$

مسئله را در دو مرحله حل می کنیم

$0<t<0.05$

یکبار $0<t$ و $t>0$ (قبل از زدن کلید)



مسئله را حل می کنیم

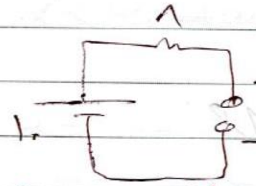
پس در (0.5s) $V_c$ مقدار $V_c$ را تعیین می کنیم (قبل از زدن کلید)

$V_c(t) = V_c(\infty)(1-e^{-t/\tau})$

پس $V_c(0.8s)$ را بدست می آوریم

$V_c(\infty)$ بدست آوردن

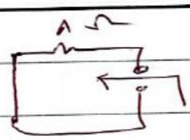
بعد از زدن کلید



$V_c(\infty) = 10V$

اگر وسط کار در ۰/۰۵ اتفاقی نمی افتاد در بینهایت $V_c(\infty) = 10$ می شد.

بدست آوردن $\tau$ :



$\tau = R_{eq} \cdot C = 8 \times 0.05 = 0.4$

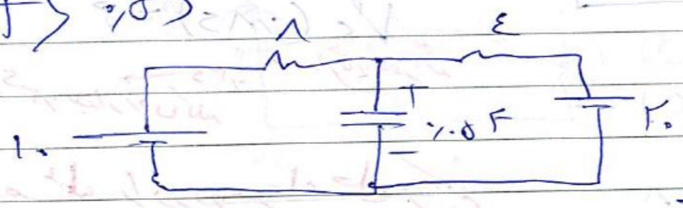
$$\boxed{V_C(t) = 10(1-e^{-2.5t})}$$

$V_C(0.4) = 10(1-e^{-1}) = \boxed{6.32}$ ✓

در لحظه ۰٫۵ :

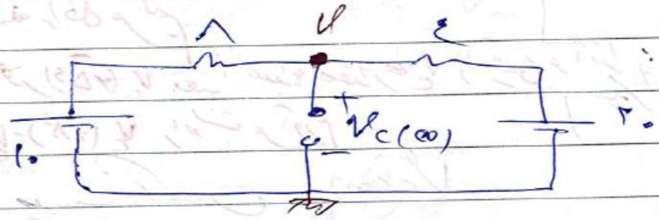
$V_C(0.5) = 10(1-e^{-1.25}) = 7.13$

$t > 0.5 s$



$$V_C(t) = (V_C(0.5) - V_C(\infty))\, e^{-\frac{(t-0.5)}{\tau}} + V_C(\infty)$$

بدست آوردن $V_C(\infty)$ :



$$\frac{V-10}{8} + \frac{V-20}{4} = 0 \rightarrow V = 16.66\, V$$

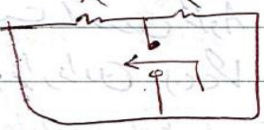
$$V_C(\infty) = 16.66\, V$$

تحلیل مدار A

نکته: همیشه $V_C(0^-) = V_C(0^+)$ یا $V_C(t_0^-) = V_C(t_0^+)$

Date: ........................ Subject: ........................

به دست آوردن $\tau$ :



$$R_{eq} = 4 \| 8 = \frac{8}{3}\ \Omega$$

$$\tau = R.C = \frac{8}{3} \times 0.5 = \frac{4}{3} \Rightarrow \frac{1}{\tau} = 0.75$$

$$V_C(t) = (7.13 - 16.66)e^{-0.75(t-0.5)} + 16.66$$

$$\boxed{V_C(t) = 16.66 - 9.53\, e^{-0.75(t-0.5)}}$$

$$V_C(0.5^+ s) = 16.66 - 9.53\, e^{\cancel{-0.75}\,0} = 7.13$$

خازن در مقابل تغییرات ولتاژ مخالفت می‌کند.

$$\boxed{V_C(0.8\, s) = 15.45}$$

۰٫۲۶ بعد از زدن کلید

* به نام خدا * هفته هفتم تحلیل